Seat No. _______________

HA-003-1016002

B. Sc. (Sem. VI) (CBCS) Examination

April - 2023

Mathematics : Paper - M - 09 (A)

(Mathematical Analysis - 2 & Abstract Algebra - 2)

Faculty Code : 003

Subject Code : 1016002

Time : $2\frac{1}{2}$ Hours / Total Marks : **70**

Instructions : (1) All questions are compulsory.
(2) Write answer of each question in your main answer sheet.

1 (a) Answer the following questions briefly : **4**

(1) Define Open Cover.

(2) Define: Countable set.

(3) Determine whether the subset $\{1, 2, 3\}$ of metric space R is compact or not.

(4) Define Connected set.

(b) Attempt any **one** out of two : **2**

(1) Show that subset $R - \{1\}$ is not connected.

(2) State and prove Heine-Borel theorem.

(c) Attempt any **one** out of two : **3**

(1) If A and B are compact subset of metric space X, then show that $A \cup B$ is also compact.

(2) If F is a closed subset of metric pace X and K is a compact subset of X. Then prove that $F \cap K$ is also compact.

(d) Attempt any **one** out of two : **5**

(1) Prove that continuous image of a connected set is connected in metric space.

(2) Prove that every open interval of metric space R is an open set.

2 (a) Answer the following questions briefly : **4**

(1) Define Laplace Transform.

(2) Find $L^{-1}\left(\frac{1}{s}\right)$.

(3) Find $L(\sinh t)$.

(4) Show that $L\left(t^{-1/2}\right) = \sqrt{\frac{\pi}{s}}$, where $s > 0$.

(b) Attempt any **one** out of two : **2**

(1) Find $L^{-1}\left(\frac{2s+6}{s^2+4}\right)$.

(2) Find $L\left(e^{-2t} \cdot \sin 5t\right)$.

(c) Attempt any **one** out of two : **3**

(1) Prove that $L^{-1}\left(\frac{s}{(s+a)^2}\right) = e^{-at}(1-at)$.

(2) If $L\{f(t)\} = \overline{f}(s)$ then prove that

$$L\left\{e^{at} f(bt)\right\} = \frac{1}{b}\overline{f}\left(\frac{s-a}{b}\right).$$

(d) Attempt any **one** out of two : **5**

(1) If $f(t) = t, 0 < t < 4$

$= 5, t > 4$ then find $L\{f(t)\}$.

(2) Prove that $L^{-1}\left(\frac{1}{\left(s^2+a^2\right)^2}\right) = \frac{1}{2a^3}(\sin at - at\cos at)$.

3 (a) Answer the following questions briefly : **4**

(1) Find $L\left(te^{t}\right)$.

(2) Write convolution theorem.

(3) Find $L(t\sin t)$.

(4) Find $L\left(\frac{\sin t}{t}\right)$.

(b) Attempt any **one** out of two : **2**

(1) If $L\{f(t)\} = \overline{f}(s)$ then prove $L\left\{\frac{f(t)}{t}\right\} = \int_s^{\infty} \overline{f}(s)$.

(2) Prove that $L\left\{te^{2t} \cdot \cos 2t\right\} = \frac{s^2 - 4s - 5}{\left(s^2 - 4s + 13\right)^2}$.

(c) Attempt any **one** out of two : **3**

(1) Prove that $L\left\{\frac{e^{-at} - e^{-bt}}{t}\right\} = \log\left(\frac{s+a}{s+b}\right)$.

(2) Prove that $L^{-1}\left(\frac{1}{s(s-1)}\right) = e^t - 1$.

(d) Attempt any **one** out of two : **5**

(1) Prove that $L^{-1}\left\{\frac{s^2 - a^2}{\left(s^2 + a^2\right)^2}\right\} = t\cos at$.

(2) Using convolution theorem, prove

$$L^{-1}\left\{\frac{1}{s\left(s^2+4\right)}\right\} = \frac{1}{4}(1 - \cos 2t).$$

4 (a) Answer the following questions briefly : **4**

(1) Define Homomorphism.
(2) Define Commutative Ring.
(3) Define Field.
(4) Define Division Ring.

(b) Attempt any **one** out of two : **2**

(1) Let $\phi : (G, *) \to (G', \Delta)$ is Homomorphism. If $H' \leq G'$ then prove $\phi^{-1}(H) \leq G$.

(2) If $\phi : (G, *) \to (G', \Delta)$ is a Homomorphism. Then $\phi(e) = e'$ where e and e' are identity elements of G and G' respectively.

(c) Attempt any **one** out of two : **3**

(1) Let I be an Ideal of a ring R with unity then prove that $I = R$ if $1 \in I$.

(2) Prove that a field has no proper ideal.

(d) Attempt any **one** out of two : **5**

(1) Prove that a commutative ring with unity is a field if it has no proper ideal.

(2) Show that $R = \left\{a + b\sqrt{2} \,/\, a, b \in Z\right\} \neq 0$ is a ring with respect to usual addition and multiplication.

5 (a) Answer the following questions briefly : **4**

(1) Define Monic Polynomial.

(2) If polynomial $f = (0, 2, 3, 5, 0, 0, ...)$ then find degree of f.

(3) Define Quadratic polynomial.

(4) Define Constant polynomial.

(b) Attempt any **one** out of two : **2**

(1) Find inverse of quaternion $1 + i + j + k$.

(2) Prove that the degree of a unit element in $D[X]$ is always zero.

(c) Attempt any **one** out of two : **3**

(1) State and prove remainder theorem of polynomials.

(2) In $R[x], f(x) = 4x^4 - 3x^2 + 1$ is divided by $g(x) = x^3 - 2x + 1$ then find quotient $q(x)$ and remainder $r(x)$.

(d) Attempt any **one** out of two : **5**

(1) State and prove division algorithm for polynomials.

(2) In usual notation, let $f, g \in D[x] - \{0\}$ then prove that $[fg] = [f] + [g]$.